Math 2135 - Assignment 7

Due October 19, 2018

(1) Explain whether the following are true or false (give counter examples if possible):
 (a) Every vector space is a subspace of itself.
 (b) Each plane in $\mathbb{R}^3$ is a subspace.
 (c) Let U be a subspace of a vector space V. Any linear combination of vectors of U is also in V.
 (d) Let $v_1, \ldots, v_n$ be in a vector space V. Then $\operatorname{Span}(v_1, \ldots, v_n)$ is the smallest subspace of V containing $v_1, \ldots, v_n$.

(2) Explain whether the following are true or false (give counter examples if possible):
 (a) Vectors v_1, v_2, v_3 are linearly dependent if v_2 is a linear combination of v_1, v_3.
 (b) A subset $\{v\}$ of a vector space is linearly dependent iff $v = 0$.
 (c) Two vectors in $\mathbb{R}^3$ cannot span all of $\mathbb{R}^3$.
 (d) There exist four vectors in $\mathbb{R}^3$ that are linearly independent.

(3) Let $v_1, \ldots, v_n$ be linearly independent in a vector space V. Show that no vector in $\operatorname{Span}(v_1, \ldots, v_n)$ can be expressed by two different linear combinations.

Hint: Use contraposition. Assume some vector u can be written as linear combination with distinct lists of coefficients $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$. Show that $v_1, \ldots, v_n$ is linearly dependent.

(4) Which of the following are bases of $\mathbb{R}^3$? Why or why not?

$$A = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}), B = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}0\\-1\\4\end{bmatrix}), C = (\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix})$$

(5) The column space $\operatorname{Col} A$ of a matrix A is the span of the columns of A.

Give a basis for $\operatorname{Nul} A$ and a basis for $\operatorname{Col} A$ over $\mathbb{R}$ for

$$A = \begin{bmatrix} 0 & 2 & 0 & 3 \\ 1 & -4 & -1 & 0 \\ -2 & 6 & 2 & -3 \end{bmatrix}.$$

(6) Give 2 different bases for

$$U = \operatorname{Span}\{\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}3\\-1\\0\end{bmatrix}, \begin{bmatrix}-1\\3\\4\end{bmatrix}\} \text{ over } \mathbb{R}.$$

(7) Show that $\cos x, \cos 2x$ are linearly independent in the vector space of functions $\mathbb{R}^{\mathbb{R}}$ over $\mathbb{R}$.

(8) Consider the vector space of functions $V = \operatorname{Span}\{\cos x, 2\cos x, \cos 2x, 3\cos 2x\}$ over $\mathbb{R}$. Give a basis for V.