Math 2135 - Assignment 6

Due October 12, 2018

(1,2) Let V be a vector space over the field F with zero vector $\mathbf{0}$, let $v \in V$ and $c \in F$. Prove that

$$cv = \mathbf{0} \text{ iff } c = 0 \text{ or } v = \mathbf{0}.$$

Show the implication $\Leftarrow$ as exercise (1) and $\Rightarrow$ as exercise (2). State the axioms that you are using.

(3) Let $V := \{f \colon \mathbb{R} \to \mathbb{R}\}$ be the vector space of all functions over $\mathbb{R}$.
  (a) Is $\{f \colon \mathbb{R} \to \mathbb{R} \mid f(0) = 1\}$ a subspace of V?
  (b) Is $\{f \colon \mathbb{R} \to \mathbb{R} \mid f(1) = 0\}$ a subspace of V?
  (c) Is $\{f \colon \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous}\}$ a subspace of V?

(4) Let $v_1, \ldots, v_n$ be vectors in a vector space V over some field F. Show that $U := \operatorname{Span}\{v_1, \ldots, v_n\}$ is a subspace of V.

(5) Let F be a field, and let $A \in F^{m \times n}$. Prove that the nullspace of A is a subspace of F^n.

(6) Let V be a vector space over F, and let U_1, U_2 be subspaces of V. Show that $U_1 \cap U_2$ is a subspace of V.

(7) Which of the following sets of vectors is linearly independent?
  (a) $\begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$
  (b) $\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 11 \\ 0 \end{bmatrix}$

(8) Are the functions $1, x, x^2$ in the vector space $V = \{f \colon \mathbb{R} \to \mathbb{R}\}$ linearly independent?