Math 2135 - Assignment 5

Due October 5, 2018

(1) (a) Which of the vectors u, v, w are in $\operatorname{Nul} A$?

$$u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \quad w = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 2 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{bmatrix}$$

(b) Solve $Ax = 0$ and give the solution in parametric vector form.

(c) Find vectors $v_1, \dots, v_k \in \mathbb{R}^4$ such that $\operatorname{Nul} A = \operatorname{Span}\{v_1, \dots, v_k\}$.

(2) Determine all the vectors in $\mathbb{R}^3$ that are orthogonal to $\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ without using the cross product. What is this set of vectors geometrically?

(3) Use row reduction to solve the following linear system over $\mathbb{Z}_2$ (for brevity we write $0, 1$ for the classes $[0], [1]$, respectively):

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \cdot x = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

(4) Let F be a field. Show that $F^{2\times 2}$ forms a vector space with $+$ the sum of matrices and $\cdot$ the multiplication of a matrix by a scalar. In particular, determine what is the zero vector and what is the additive inverse of a matrix in $F^{2\times 2}$.

(5) Prove or disprove:

(a) $U = \{\begin{bmatrix} t \\ 2t \end{bmatrix} \mid t \in \mathbb{R}\}$ is a vectorspace over $\mathbb{R}$ with componentwise addition and scalar multipes;

(b) $V = \{\begin{bmatrix} 1+t \\ 2t \end{bmatrix} \mid t \in \mathbb{R}\}$ is a vectorspace over $\mathbb{R}$ with componentwise addition and scalar multipes.

Hint: For proving that V is a vectorspace, check all the axioms. For showing that V is not a vectorspace, find one axiom which is not satisfied.