Math 2135 - Assignment 3

Due September 21, 2018

Solve all systems of linear equations by row reduction (Gaussian elimination) and determine the pivot columns.

(1) Is the multiplication of matrices commutative whenever it is defined? Give a proof or a counterexample.

(2) For $n \in \mathbb{N}$, the $n \times n$ **identity matrix** I_n has entries $e_{ij} = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{else,} \end{cases}$ for $1 \leq i, j \leq n$.

Let $m, n \in \mathbb{N}$ and $A \in \mathbb{R}^{m \times n}$. Show that

$$I_m \cdot A = A = A \cdot I_n.$$

(3) Do the following 4 planes intersect in a point? Which?

$$\begin{aligned} x + 5y + 3z &= 16 \\ 2x + 10y + 8z &= 34 \\ 4x + 20y + 15z &= 67 \\ x + 6y + 5z &= 21 \end{aligned}$$

(4) Add an equation of a line to the equation

$$2x + 3y = 4$$

such that the resulting system has (a) no solution, (b) exactly one solution, (c) infinitely many solutions.

(5) Solve the system of linear equations with augmented matrix

$$\begin{bmatrix} 0 & 0 & 1 & 4 \\ 2 & -4 & 1 & 0 \\ -3 & 6 & 2 & 7 \end{bmatrix}$$

(6) Solve the system of linear equations with augmented matrix

$$\begin{bmatrix} 2 & 8 & 6 & 1 & 1 \\ 1 & 4 & 2 & 0 & 1 \\ 2 & 8 & 10 & 3 & -1 \end{bmatrix}$$

(7) Let

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 8 \\ -16 \\ 12 \end{bmatrix}, \quad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Solve the equations $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$. Express both solution sets in parametric vector form. Give a geometric description of the solution sets.

(8) Are the following true or false? Explain your answers.
   (a) Any system of linear equations with strictly less equations than variables has infinitely many solutions.
   (b) Different sequences of elementary row reductions can transform one matrix to distinct matrices in row echelon form.
   (c) A consistent system has exactly one solution.
   (d) There exist inconsistent homogenous systems.
   (e) If a homogenous system has strictly less equations than variables, then it has infinitely many solutions.