Math 2135 - Assignment 2

Due September 14, 2018

Vectors are denoted in boldfaced to better distinguish them from scalars. More challenging problems are marked by *.

(1) Find the angle θ between the following vectors $\mathbf{u}, \mathbf{v}$:
 (a) $\mathbf{u} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$,
 (b) $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$.

(2) Give 3 vectors in different directions that are orthogonal to $\mathbf{v} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$.

(3) [1, Ex. 1.2.16] Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$. Prove that

$$\mathbf{u} \cdot \mathbf{v} = \frac{1}{4}(||\mathbf{u} + \mathbf{v}||^2 - ||\mathbf{u} - \mathbf{v}||^2).$$

So the norm of vectors can be expressed using the dot product, and conversely the dot product can be expressed using the norm.

(4) Compute the projection $\mathrm{proj}_{\mathbf{v}}\mathbf{u}$ for the vectors in (1).

(5) Vectors $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ are called the **standard unit vectors** in $\mathbb{R}^3$.

What are the projections of an arbitrary vector $\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ onto $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, respectively.

(6) Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ with $\mathbf{u}$ not the zero vector.
 (a) Assume $\mathbf{u}, \mathbf{v}$ are orthogonal. What is the projection $\mathrm{proj}_{\mathbf{v}}\mathbf{u}$?
 (b) Assume $\mathbf{v} = c\mathbf{u}$ is a scalar multiple of $\mathbf{u}$. What is the projection $\mathrm{proj}_{\mathbf{v}}\mathbf{u}$?

(7) If possible, compute the following for the matrices

$$A = \begin{bmatrix} 5 & -1 \\ -3 & 4 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & -4 \\ 3 & -1 & 1 \end{bmatrix}, C = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix}$$

 (a) AB
 (b) BA
 (c) BC
 (d) $A + C$
 (e) $BA + C$

(8) Let $A \in \mathbb{R}^{m \times n}$ and $B, C \in \mathbb{R}^{n \times p}$. Show the distributive law

$$A(B + C) = AB + AC.$$

References

[1] S. Andrilli and D. Hecker. Elementary Linear Algebra. Elsevier, 5th edition, 2016.