Math 2135 - Assignment 1

Due September 7, 2018

Vectors are denoted in boldfaced to better distinguish them from scalars. More challenging problems are marked by *.

(1) Show that the points $\begin{bmatrix} -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ are the corners of an isosceles triangle. Is this triangle equilateral?

(2) (a) Give the vector in direction of $\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$ of length 4.

(b) Find the point that is two-thirds of the distance from $\begin{bmatrix} 4 \\ -2 \end{bmatrix}$ to $\begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

(3) Give the parametric form for the lines through the following points. Does either one contain the point $\begin{bmatrix} 8 \\ -3 \\ 6 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

(4) (a) Describe the points of the yz-plane in $\mathbb{R}^3$.

(b*) Describe the plane through the points $\begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ in parametric form.

Hint: Use 2 parameters.

(5) The vector $\mathbf{0} := [0, \ldots, 0]$ in $\mathbb{R}^n$ with all entries 0 is called the **zero vector** in $\mathbb{R}^n$.

Let $\mathbf{v} \in \mathbb{R}^n$. Show that $||\mathbf{v}|| = 0$ if and only if $\mathbf{v} = \mathbf{0}$.

You have to prove the following 2 directions:

(a) Assume $\mathbf{v} = \mathbf{0}$. Show $||\mathbf{v}|| = 0$.

(b*) Assume $||\mathbf{v}|| = 0$. Show $\mathbf{v} = \mathbf{0}$.

(6) The **dot product** of $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ is defined as $\mathbf{u} \cdot \mathbf{v} = \sum_{i=1}^n u_i v_i$.

Let $\mathbf{u}, \mathbf{v}, \mathbf{w} \in \mathbb{R}^n$. Show that

$$(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$$

(7) Let $n > 1$. Show that for $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ in general it is not true that $(\mathbf{u} \cdot \mathbf{v})\mathbf{w} = \mathbf{u}(\mathbf{v} \cdot \mathbf{w})$.

Give a concrete counter-example.

(8) Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ with $\mathbf{u} \neq \mathbf{0}$. Does $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$ imply $\mathbf{v} = \mathbf{w}$?

Prove it or give a counter-example.