Math 2135 - Assignment 0

Due August 31, 2018

These problems will not be graded. They are meant to revise some notions from Math 2001, Discrete Mathematics, that will be used in this course. If necessary, please look up Section 1.1 on sets and Chapter 2 on logic in

> Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free on `http://www.people.vcu.edu/~rhammack/BookOfProof/`

(1) (a) Give two points on the line in $\mathbb{R}^2$ that is described by the equation $x - 2y = 3$.
  (b) Find the intersection of the lines given by $2x - 3y = 4$ and by $3x + 2y = 1$.

(2) The points on the x-axis in $\mathbb{R}^2$ are all of the form $(x, 0)$ for $x \in \mathbb{R}$. Hence in set builder notation the x-axis is the set

$$\{(x, 0) \mid x \in \mathbb{R}\}.$$

Describe the following using set builder notation:
  (a) $A =$ the set of points in $\mathbb{R}^2$ on the line through $(2, 3)$ that is parallel to the y-axis
  (b) $B =$ the set of points $(x, y) \in \mathbb{R}^2$ on the line through $(1, 2)$ and $(3, 4)$
  (c) $C =$ the set of points in $\mathbb{R}^2$ that lie on a circle with center $(0, 0)$ and radius 2

(3) Are the following true or false? Explain why. (A, B, C are arbitrary statements.)
  (a) Assume A implies B and B implies C. Then A implies C.
  (b) A implies B and B implies A means that A is true whenever B is true, and A is false whenever B is false.
  (c) n is an even integer $\Leftrightarrow n + 1$ is an odd integer
  (d) For all $x, y \in \mathbb{R}$, we have $xy = 0$ iff $x = 0$ and $y = 0$.

(4) Give the negations of the following statements:
  (a) $A \Rightarrow B$
  (b) If you do well on the homework, you'll pass the class.
  (c) $A \Leftrightarrow B$
  (d) $x \in \mathbb{R}$ has an inverse if and only if $x \neq 0$.

(5) Which of the following are true? Explain why or why not.
  (a) $\forall x \in \mathbb{R}\ \forall y \in \mathbb{R} : x + y = 0$
  (b) $\forall x \in \mathbb{R}\ \exists y \in \mathbb{R} : x + y = 0$
  (c) $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R} : x + y = 0$
  (d) $\exists x \in \mathbb{R}\ \exists y \in \mathbb{R} : x + y = 0$