Math 2130 Fall 2021 - Review for Finals

Numbers refer to sections in Lay et al, Linear algebra and its applications, ed. 5.

1. **Systems of linear equations.**
   (1) coefficient and augmented matrix (1.4)
   (2) solving a linear system by row reduction, pivot columns, free variables, solution in parametrized vector form (1.2)
   (3) solutions of homogenous vs. inhomogenous systems (1.5)
   (4) least squares solutions (6.5)

2. **Vector spaces.**
   (1) vector spaces and examples: tuples, functions, polynomials P_n (4.1)
   (2) subspaces: definition and examples (span, null space) (4.1)

3. **Basis of vector spaces.**
   (1) dimension, Basis Theorem (4.5)
   (2) reduce a spanning set to a basis (2.8, 4.3), extend a linear independent set to a basis (4.5)
   (3) bases and dimension for column space, row space, null space of a matrix (2.8, 4.3)
   (4) coordinates with respect to a basis B (2.9, 4.4)
   (5) change of coordinate matrix $P_{B \Leftarrow C}$ for bases B and C (4.7)
   (6) orthogonal basis, coordinates via dot product (6.2), orthogonal projection (6.3), Gram–Schmidt process (6.4)

4. **Matrices.**
   (1) matrix product and composition of linear maps (2.1)
   (2) inverse matrices and their properties, Invertible Matrix Theorem (2.2, 2.3)
   (3) rank of a matrix (4.6)
   (4) inverse matrix via row reduction (2.2)
   (5) formula for inverse of 2×2-matrix (2.2)
   (6) determinant via cofactor expansion (3.1 and via row reduction (3.2)
   (7) eigenvalues and eigenvectors of matrices (5.1), characteristic polynomials (5.2)
   (8) diagonalizing matrices, powers of matrices (5.3)

5. **Linear maps.**
   (1) a linear map is determined by its images on a basis (1.8)
   (2) matrix $T_{B\leftarrow C}$ of a linear map f with respect to bases B, C, standard matrix $T_{E\leftarrow E}$ (for standard basis E of $\mathbb{R}^n$) (1.9, 4.7)
   (3) matrix for rotation, reflection in $\mathbb{R}^2$ and $\mathbb{R}^3$ (1.9)
   (4) injective, surjective, bijective linear maps and connections with kernel, range (4.2)
   (5) isomorphism between vector spaces, n-dimensional vector space is isomorphic to $\mathbb{R}^n$ (4.4)