Math 2130 - Practice Final

December 6-8, 2021

(1) Let $B = (\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}3\\4\end{bmatrix})$.

(a) Why is B a basis of $\mathbb{R}^2$?

(b) Give change of coordinates matrices $P_{E\leftarrow B}$ (for changing B-coordinates into coordinates w.r.t. the standard basis E) and $P_{B\leftarrow E}$.

(c) Compute the coordinates $[x]_B$ for $x = \begin{bmatrix}2\\3\end{bmatrix}$.

(2) Let $B = (b_1, b_2)$ as in the previous problem. Let $h\colon \mathbb{R}^2 \to \mathbb{R}^2$ be linear such that $[h(b_1)]_E = \begin{bmatrix}-1\\1\end{bmatrix}, [h(b_2)]_E = \begin{bmatrix}0\\1\end{bmatrix}$.

(a) Give the standard matrix $T_{E\leftarrow E}$ of h w.r.t. the standard basis.

(b) Compute $h(\begin{bmatrix}1\\1\end{bmatrix})$.

(3) Let

$$A = \begin{bmatrix}1 & -2 & 3\\ -2 & 0 & 1\\ 3 & -2 & 2\end{bmatrix}$$

(a) Is the mapping $h\colon \mathbb{R}^3 \to \mathbb{R}^3$, $x \mapsto Ax$, injective, surjective, bijective?

(b) Give bases for null space, row space, column space of A.

(4) Let A be the standard matrix for the rotation r of $\mathbb{R}^2$ by angle φ counterclockwise around the origin. What are the eigenvalues and eigenvectors of A? Can A be diagonalized over the reals?

(5) Diagonalize A if possible. Also compute $\det A$. Is A invertible?

$$A = \begin{bmatrix}1 & 2 & 3\\ 3 & 0 & 3\\ 0 & 0 & -2\end{bmatrix}$$

(6) Compute the inverse if possible:

$$A = \begin{bmatrix}1 & -2\\ -2 & 4\end{bmatrix}, \quad B = \begin{bmatrix}1 & 2 & 4\\ 0 & 0 & -1\\ 1 & -1 & 0\end{bmatrix}$$

(7) Let $h\colon V \to W$ be a linear map, let $v_1, \ldots, v_k \in V$ such that $h(v_1), \ldots, h(v_k)$ are linearly independent. Show that $v_1, \ldots, v_k$ are linearly independent.