Math 2130 - Assignment 10

Due November 5, 2021

Problems 1-7 are review material for the second midterm on November 3. Solve them before Wednesday!

(1) Let $T\colon P_2 \to \mathbb{R}, p \mapsto p(3)$, be the map that evaluates a polynomial p at $x = 3$.
   (a) Show that T is linear.
   (b) Determine the kernel of T, that is, $\{p \in P_2 \ : \ T(p) = 0\}$, and the image of T, that is, $T(P_2)$.
   (c) Is T injective, surjective, bijective?

(2) Let $B = (b_1, b_2)$ with $b_1 = \begin{bmatrix} -5 \\ 11 \\ 5 \end{bmatrix}, b_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$ and $C = (\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix})$ be bases of a subspace H of $\mathbb{R}^3$.
   (a) Compute the coordinates $[b_1]_C$ and $[b_2]_C$.
   (b) What is the change of coordinate matrix $P_{C\leftarrow B}$?
   (c) What is the change of coordinate matrix $P_{B\leftarrow C}$?

(3) Let $C = (1 + t, t + t^2, 1 + t^2)$ be a basis for P_2. Compute the coordinates $[p]_C$ for $p = 2 + t^2$.

(4) (a) If A is a 3×4-matrix, what is the largest possible rank of A? What is the smallest possible dimension of Nul A?
   (b) If the nullspace of a 4×6-matrix B has dimension 3, what is the dimension of the row space of B?
   (c) Give two 3×3-matrices with determinant 6.

(5) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} u & v \\ w & x \end{bmatrix}$. Show

$$\det(AB) = \det(A)\det(B).$$

(6) Let $A \in \mathbb{R}^{n\times n}$ and $\lambda \in \mathbb{R}$. Is

$$H = \{\mathbf{x} \in \mathbb{R}^n \ : \ A\mathbf{x} = \lambda\mathbf{x}\}$$

a subspace of $\mathbb{R}^n$? Which conditions for a subspace are fulfilled by H?

(7) For which $\mu \in \mathbb{R}$ has the matrix

$$B = \begin{bmatrix} 6-\mu & 2 \\ -6 & -1-\mu \end{bmatrix}$$

a determinant $\det B = 0$?

(8) Let

$$A = \begin{bmatrix} 6 & 2 \\ -6 & -1 \end{bmatrix}.$$

   (a) Compute the matrices $A - 2I$, $A - 3I$, and $A - I$.
   (b) Find all $\mathbf{x} \in \mathbb{R}^3$ such that $A\mathbf{x} = 2\mathbf{x}$. Give the parametrized vector form for the solution set.
   Hint: $A\mathbf{x} = 2\mathbf{x}$ iff $A\mathbf{x} = 2I\mathbf{x}$ iff $(A - 2I)\mathbf{x} = \mathbf{0}$.
   (c) Find all $\mathbf{x} \in \mathbb{R}^3$ such that $A\mathbf{x} = 3\mathbf{x}$. Give the parametrized vector form.
   (d) Find all $\mathbf{x} \in \mathbb{R}^3$ such that $A\mathbf{x} = \mathbf{x}$. Give the parametrized vector form.