Math 2130 - Assignment 8

Due October 22, 2021

(1) Give 2 different bases for

$$H = \text{Span}\{\begin{bmatrix}1\\1\\2\end{bmatrix}, \begin{bmatrix}3\\-1\\0\end{bmatrix}, \begin{bmatrix}-1\\3\\4\end{bmatrix}\}$$

(2) Let $B = (b_1, \ldots, b_n)$ be a basis for a vector space V and consider the coordinate mapping $V \to \mathbb{R}^n,\ x \mapsto [x]_B$.

(a) Show that $[c \cdot x]_B = c[x]_B$ for all $x \in V, c \in \mathbb{R}$.

(b) Show that the coordinate mapping is onto $\mathbb{R}^n$.

(3) Let $B = (\begin{bmatrix}1\\-2\end{bmatrix}, \begin{bmatrix}-3\\4\end{bmatrix})$ be a basis of $\mathbb{R}^2$.

(a) Give the change of coordinates matrix $P_{E\leftarrow B}$ from B to the standard basis $E = (e_1, e_2)$ and $P_{B\leftarrow E}$.

(b) Find vectors $u, v \in \mathbb{R}^2$ with $[u]_B = \begin{bmatrix}0\\1\end{bmatrix}, [v]_B = \begin{bmatrix}3\\2\end{bmatrix}$.

(c) Compute the coordinates relative to B of $w = \begin{bmatrix}-2\\4\end{bmatrix}$ and $x = \begin{bmatrix}1\\0\end{bmatrix}$.

(4) Let $B = (1, t, t^2)$ and $C = (1, 1+t, 1+t+t^2)$ be bases of $\mathbb{P}_2$.

(a) Determine the polynomials p, q with $[p]_B = \begin{bmatrix}3\\0\\-2\end{bmatrix}$ and $[q]_C = \begin{bmatrix}3\\0\\-2\end{bmatrix}$.

(b) Compute $[r]_B$ and $[r]_C$ for $r = 3 + 2t + t^2$.

(5) Let $\mathbf{b}_1 = \begin{bmatrix}1\\2\\-1\end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix}1\\1\\3\end{bmatrix}, \mathbf{b}_3 = \begin{bmatrix}1\\2.5\\-5\end{bmatrix}$.

(a) Find vectors $\mathbf{u}_1, \ldots, \mathbf{u}_k$ such that $(\mathbf{b}_1, \mathbf{b}_2, \mathbf{u}_1, \ldots, \mathbf{u}_k)$ is a basis for $\mathbb{R}^3$.

(b) Find vectors $\mathbf{v}_1, \ldots, \mathbf{v}_\ell$ such that $(\mathbf{b}_3, \mathbf{v}_1, \ldots, \mathbf{v}_\ell)$ is a basis for $\mathbb{R}^3$.

Prove that your choices for (a) and (b) form a basis.

(6) Let

$$A = \begin{bmatrix}-5 & 8 & 0 & -17 & -2\\ 3 & -5 & 1 & 5 & 1\\ 11 & -19 & 7 & 1 & 3\\ 7 & -13 & 5 & -3 & 1\end{bmatrix}.$$

Find bases and dimensions for $\text{Nul}\, A$, $\text{Col}\, A$, and $\text{Row}\, A$, respectively.

(7) A 25×35 matrix A has 20 pivots. Find $\dim \text{Nul}\, A$, $\dim \text{Col}\, A$, $\dim \text{Row}\, A$, and $\text{rank}\, A$.

(8) True or false? Explain.

(a) If B is an echelon form of a matrix A, then the pivot columns of B form a basis for the column space of A.

(b) If B is an echelon form of a matrix A, then the nonzero rows of B form a basis for the row space of A.

(c) A basis of B is a set of linear independent vectors in V that is as large as possible.

(d) If $\dim V = n$, then any n vectors that span V are linearly independent.

(e) Every 2-dimensional subspace of $\mathbb{R}^2$ is a plane.