Math 2130 - Assignment 5

Due October 1, 2021

Problems 1-5 are review material for the first midterm on September 29. Solve them before Wednesday!

(1) Let
$$A = \begin{bmatrix} 0 & 3 & 1 & 2 \\ 1 & 4 & 0 & 7 \\ 2 & -1 & -3 & 8 \end{bmatrix}, b = \begin{bmatrix} 6 \\ 5 \\ -8 \end{bmatrix}$$

  (a) Give the solution for $Ax = b$ in parametrized vector form.
  (b) Give vectors that span the null space of A.

(2) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with
$$T(\begin{bmatrix} 1 \\ 2 \end{bmatrix}) = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \text{ and } T(\begin{bmatrix} 3 \\ 4 \end{bmatrix}) = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}.$$
What is the standard matrix of T?

(3) Let $T\colon \mathbb{R}^n \to \mathbb{R}^n, x \mapsto Ax$, be a surjective linear map. Show that T is injective as well.

(4) True or false? Explain your answer.
  (a) If $Ax = b$ is inconsistent for some vector b, then A cannot have a pivot in every column.
  (b) If vectors $\mathbf{v}_1, \mathbf{v}_2$ are linearly independent and $\mathbf{v}_3$ is not in the span of $\mathbf{v}_1, \mathbf{v}_2$, then $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ is linear independent.
  (c) The range of $T\colon \mathbb{R}^n \to \mathbb{R}^m, x \mapsto Ax$, is the span of the columns of A.

(5) (a) Give examples of square matrices A, B such that neither A nor B is 0 (the matrix with all entries 0) but $AB = 0$.
  (b) If the first two columns of a matrix B are equal, what can you say about the columns of AB?
  (c) We can view vectors in $\mathbb{R}^n$ as $n \times 1$ matrices. For $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ compute $\mathbf{u}^T \cdot \mathbf{v}$ and $\mathbf{u} \cdot \mathbf{v}^T$. Interpret the results.

(6) Prove for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc \neq 0$ that
$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

(7) Are the following invertible? Give the inverse if possible.
$$A = \begin{bmatrix} 2 & 1 \\ 4 & -9 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -6 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

(8) A **diagonal matrix** A has all entries 0 except on the diagonal, that is,
$$A = \begin{bmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & a_{nn} \end{bmatrix}.$$
Under which conditions is A invertible and what is A^{-1}?