Math 2001 - Assignment 13

Due April 24, 2020

(1) Try to you find an inverse for $f\colon \mathbb{R} \to \mathbb{R}^+,\ x \mapsto e^{x^3+1}$. Is f bijective?
(2) Find the inverse for $f\colon \mathbb{R}^2 \to \mathbb{R}^2,\ (x,y) \mapsto (3x+y, x-2y)$.
(3) Let c be the function on the power set of $\mathbb{Z}$ that maps every set to its complement, i.e.,
$$c\colon P(\mathbb{Z}) \to P(\mathbb{Z}), X \mapsto \bar{X}.$$
Determine c^{-1}.
(4) Give an explicit bijection $f\colon [0,1] \to (0,1)$. Show that your function f is bijective.
(5) Show that the set $A = \{X \subseteq \mathbb{N} \ :\ X \text{ is finite}\}$ of all finite subsets of $\mathbb{N}$ is countable.
(6) Show that $|\mathbb{R} \times \mathbb{R}| = |\mathbb{R}|$.
Hint: Give a bijection $(0,1)^2 \to (0,1)$ first.