Math 2001 - Assignment 12

Due April 17, 2018

(1) (a) Give domain, codomain, and range of $f\colon \mathbb{Z} \to \mathbb{N}$, $x \mapsto x^2+1$. What is $f(3)$?
(b) Is f one-to-one, onto, bijective?
(c) Determine $f(\{2x \ : \ x \in \mathbb{Z}\})$ and $f^{-1}(\{1,2,3,\dots,10\})$.

(2) Give examples for
(a) a function $f\colon \mathbb{N} \to \mathbb{N}$ that is not injective but surjective;
(b) a function $g\colon \{1,2,3\} \to \{1,2\}$ that is neither injective nor surjective;
(c) a bijective function $h\colon \{1,2,3\} \to \{1,2\}$.

(3) Let A, B be finite sets with $|A| = |B|$, and let $f\colon A \to B$. Show that f is injective iff f is surjective.

Is this true for functions between infinite sets as well? Prove it or give counterexamples for each direction.

(4) Prove or give counterexamples:
(a) The composition of two injective functions is injective.
(b) The composition of two surjective functions is surjective.

(5) Let $f\colon A \to B, g\colon B \to C$. Show that
(a) If $g \circ f$ is injective, then f is injective.
(b) If $g \circ f$ is surjective, then g is surjective.
Hint: Use contrapositive proofs.

Give examples for f, g on $A = B = C = \mathbb{N}$ such that
(c) $g \circ f$ is injective but g is not injective;
(d) $g \circ f$ is surjective but f is not surjective.

(6) (a) Show that
$$f : \mathbb{R} - \{1\} \to \mathbb{R} - \{2\}, x \mapsto \frac{2x+1}{x-1}$$
is bijective.
(b) Determine f^{-1}.