Math 2001 - Assignment 11

Due April 10, 2020

(1) Prove or disprove that the following relations are reflexive, symmetric, antisymmetric, transitive. Which are equivalences, which partial orders?
  (a) $\neq$ on $\mathbb{Z}$
  (b) $\subseteq$ on the power set $P(A)$ of a set A

(2) Prove or disprove that the following relations are reflexive, symmetric, antisymmetric, transitive. Which are equivalences, which partial orders?
  (a) $|$ (divides) on $\mathbb{N}$
  (b) $R = \{(x, y) \in \mathbb{R} \colon |x - y| \leq 1\}$

(3) List the equivalence classes for these equivalence relations:
  (a) The relation $\sim$ on subsets A, B of $\{1, 2, 3\}$ where $A \sim B$ if $|A| = |B|$.
  (b) $R = \{(x, y) \in \mathbb{Z} \colon |x| = |y|\}$ on $\mathbb{Z}$

(4) (a) Given finite sets A and B. How many different relations are there from A to B?
  (b) How many different equivalence relations are there on $A = \{1, 2, 3\}$? Describe them all by listing the partitions of A.

(5) Let $\sim$ be an equivalence relation on a set A, let $a, b \in A$. Let $[a]$ denote the equivalence class of a modulo $\sim$. Show that

$$a \not\sim b \text{ iff } [a] \cap [b] = \emptyset.$$

(6) (a) Give the tables for addition and multiplication for $\mathbb{Z}_6$.
  (b) Dividing by $[a]$ in $\mathbb{Z}_n$ means solving an equation $[a] \cdot [x] = [1]$ for $[x]$.
  Solve $[8] \cdot [x] = [1]$ in $\mathbb{Z}_{37}$.
  Hint: Use the Euclidean algorithm to solve $8x \equiv 1 \mod 37$.