Math 2001 - Assignment 5

Due February 21, 2020

Problems 1-3 are review material for the first midterm on February 19. So you should solve them before Wednesday!

(1) Simplify:
   (a) $\bigcup_{i=0}^{4} [i, 2i+1]$
   (b) $\bigcap_{n\in\mathbb{N}} \{x \in \mathbb{Z} : x \geq n\}$

(2) (a) Is it true that for all statements P, Q

$$(P \Rightarrow Q) \wedge P = Q$$

   Prove it or give a counter-example.
   (b) Show that $\sim (P \Leftrightarrow Q) = (\sim P) \Leftrightarrow Q$

(3) Write using quantifiers and logical operations. Is the statement true? Give its negation.
   (a) The square of any real number is non-negative.
   (b) There exists an integer x such that $x^y = x$ for all integers y.
   (c) For all reals x and y we have that $xy = 0$ implies $x = 0$.

(4) How many lists of length 4 are there with entries from A,. . . , Z if
   (a) repetition is allowed,
   (b) repetition is not allowed,
   (c) repetition is not allowed and the list must contain A,
   (d) repetition is allowed and the list must contain A.

(5) [1, Section 3.3]: Exercise 2

(6) How many standard Colorado license plates (3 numbers followed by 3 letters) have at least one number or letter repeated?

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`