Math 2001 - Assignment 4

Due February 14, 2018

(1) Are the following equalities true for all statements P, Q?
  (a) $P \wedge \sim P = \text{False}$
  (b) $\sim (P \Leftrightarrow Q) \ = \ \sim P \Leftrightarrow \sim Q$

(2) [1, Section 2.7]: Exercises 4,6,9,10

(3) (a) How many different truthtables (Boolean functions) are there for 2 statements x_1, x_2? How many for k statements $x_1, \ldots, x_k$?

  (b) Let $f(x_1, x_2, x_3)$ be a Boolean function that is true for the following assignments and false otherwise.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
T	T	F	T
T	F	T	T
F	T	T	T

  Write an expression for $f(x_1, x_2, x_3)$ using only $\wedge, \vee, \sim$.

(4) Formulate the following sentences using quantifiers and logical operations. Are they true?
  (a) For all integers n we have that $n(n+1)$ is even.
  (b) If x^2 is rational, then so is x.
  (c) There exists a real number z such that $x + z = x$ for every real x.
  (d) Every real number is smaller than some integer.

(5) Negate the following sentences:
  (a) $xy = 0$ iff $x = 0$ or $y = 0$
  (b) The derivative of a polynomial function f is 0 iff f is constant.
  (c) $\exists x \in \mathbb{R} : x^2 = -1$
  (d) $\forall r \in \mathbb{R} : \sin(r\pi) = 0 \Leftrightarrow r$ is an integer

(6) Are the following sentences true? Negate them:
  (a) There exists a right triangle that is not isosceles.
  (b) $\forall x \in \mathbb{R}^+ \ \exists n \in \mathbb{N} : \frac{1}{n} \leq x$.
  (c) $\exists m \in \mathbb{N} \ \forall p \in \mathbb{N} : p \text{ prime} \Rightarrow p \leq m$

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: http://www.people.vcu.edu/~rhammack/BookOfProof/