Math 2001 - Assignment 2

Due January 31, 2020

(1) Describe the following using set builder notation (either Axiom of Specification or Replacement):
  (a) $A =$ the set of points in $\mathbb{R}^2$ on the line through $(2, 3)$ that is parallel to the y-axis
  (b) $B =$ the set of points $(x, y) \in \mathbb{R}^2$ on the line through $(1, 2)$ and $(3, 4)$
  (c) $C =$ the set of points in $\mathbb{R}^2$ that lie on a circle with center $(0, 0)$ and radius 2

(2) For $U := \{x \in \mathbb{Z} : 1 \leq x \leq 8\}, A = \{1, 2, 3, 4, 5\}$, $B = \{x \in U : x \text{ is even }\}$, and $C = \{x \in U : x \geq 4\}$ compute:
  (a) $A \cap C_U(B)$ (b) $A \cup (B \cap C)$ (c) $(A - B) \cup B$

(3) Are the following true for all sets A, B in a universe U?
  (a) $A - B = B - A$
  (b) $A \cup B \subseteq (A \cap \bar{B}) \cup (B \cap \bar{A})$

  Consider Venn diagrams first and then either write a proof that the equations hold or give an example where they fail.

(4) Show that for all sets A, B, C

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

  without Venn diagrams.

  Recall that we already showed that the lefthand side is contained in the the righthand side. So it only remains to write a proof for the converse,

$$(A \cup B) \cap C \supseteq (A \cap C) \cup (B \cap C).$$

(5) Show for all sets A, B in the universe U:

$$\overline{A \cup B} = \bar{A} \cap \bar{B} \qquad \text{(de Morgan's law)}$$

  First use Venn diagrams. Then write down a proof.

(6) Simplify the following sets and justify your answers:
  (a) $\bigcup_{n \in \mathbb{N}} (0, n]$ (b) $\bigcap_{n=1}^{3} \{nz : z \in \mathbb{Z}\}$ (c) $\bigcup_{A \in P(\mathbb{N})} A$

  In (a) we have $(0, n] = \{x \in \mathbb{R} \; : \; 0 < x \leq n\}$, the real interval from 0 to n that does not contain 0 but contains n.