Boxes

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Problem

You want to split a stack of n boxes ($n \in \mathbb{N}$), one box on top of the other, into n stacks of height 1.

- In each move, you can split a single stack, say of height n, into 2 stacks, say of heights a and b with $a + b = n$. This move has score ab.
- Then you can split one of the new stacks into two of smaller heights, say u, v, and add uv to your previous score.
- Repeat splitting single stacks and adding the product of heights of the new stacks to the previous score until you have n stacks of height 1.

What is the maximum score possible starting with a stack of height n?

- Try for small $n = 1, 2, 3, 4, \ldots$.
- What are possible strategies?
- Any conjecture for the maximum score for arbitrary n?

Small cases

height n	score(n)
1	0?
2	$1 \cdot 1 = 1$
3	$2 \cdot 1 + 1 \cdot 1 = 3$

So far the only possible strategy is to split off 1 box at a time.

4	$3 \cdot 1 +$ score(3)$ +$ score(1)
	$2 \cdot 2 +$ score(2)$ +$ score(2)

Conjecture

Any way you unstack n boxes, you always get a score of $\frac{n(n-1)}{2}$.

How'd you prove that?