HOW TO SHOW TWO SETS S AND T ARE EQUAL

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Show that $S \subseteq T$ by taking an arbitrary element $x \in S$ and argue that $x \in T$ also. Then show the converse that $T \subseteq S$ by taking an arbitrary element $x \in T$ and argue that $x \in S$ also. Since now $S \subseteq T$ and $T \subseteq S$, you actually know $S = T$.

Example 1. *Show that for all sets A, B, C the* **distributive law**

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

holds.

Proof. We have two inclusions to show:

(1) Show $(A \cup B) \cap C \subseteq (A \cap C) \cup (B \cap C)$:
   Let $x \in (A \cup B) \cap C$. Then $x \in A \cup B$ and $x \in C$ by the definition of $\cap$. Since $x \in A \cup B$, we have $x \in A$ or $x \in B$ by the definition of $\cup$. We consider these 2 cases separately:
   - Case $x \in A$: Then $x \in A$ and $x \in C$ yields $x \in A \cap C$.
   - Case $x \in B$: Then $x \in B$ and $x \in C$ yields $x \in B \cap C$.

   In either case $x \in (A \cap C) \cup (B \cap C)$.
   Hence for any element $x \in (A \cup B) \cap C$ we also have $x \in (A \cap C) \cup (B \cap C)$. This proves
   $$(A \cup B) \cap C \subseteq (A \cap C) \cup (B \cap C).$$

(2) Next show the converse $(A \cap C) \cup (B \cap C) \subseteq (A \cup B) \cap C$:
   Let $x \in (A \cap C) \cup (B \cap C)$. Then $x \in (A \cap C)$ or $x \in (B \cap C)$ ...

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