Math 2001 - Review for Midterm 2

See the handouts on the following topics on the course website. Section numbers below refer to Hammack's Book of Proof (3rd edition).

Counting.

(1) Lists: with/without repetitions, permutations - factorials (3.4), subsets - binomials (3.5), integer solutions of $x_1+x_2+\cdots+x_n = k$ (3.8)
(2) Binomial Theorem (3.6)
(3) Inclusion-Exclusion (3.7)

Modular arithmetic.

(1) Integers: divisibility, division algorithm, gcd, lcm (4.2), extended Euclidean algorithm (Sections 2,4 on handout Integers'), Bezout's identity and coefficients (Proposition 7.1)
(2) congruence modulo n (5.2)

Proof methods.

(1) direct proof (4), contrapositive proof (5), proof by contradiction (6), proof of if-and-only-if statements (7.1)
(2) proof by induction (10.1)

Some additional practice problems.

(1) (a) Compute $3-8 \mod 11$ and $16 \cdot 20 \mod 11$.
    (b) Compute $\gcd(111, 33)$ and its Bezout coefficients.
(2) Prove that $\sqrt[3]{2}$ is irrational.
(3) Let $a, b \in \mathbb{Z}$. Show that $a \equiv b \mod 6$ if and only if $a \equiv b \mod 2$ and $a \equiv b \mod 3$.
(4) Let $a \in \mathbb{Z}$. Show that $\gcd(a, a+1) = 1$.
(5) Give the first sentence (the assumptions) for the proofs of the following statements:
    (a) Let $a, b \in \mathbb{Z}$. If $a|b$ and $b|a$, then $a = b$ or $a = -b$. (direct proof)
    (b) Let $a, b \in \mathbb{Z}$. If $(a+b)^2 = a^2 + b^2$, then $a = 0$ or $b = 0$. (contrapositive proof)
    (c) Let $x, y \in \mathbb{Z}$. If $4|x^2 + y^2$, then x and y are even. (proof by contradiction)