Math 2001 Fall 2020 - Review for Final

Numbers refer to sections in Hammack, Book of Proof, ed. 3, 2018.

1. **Sets.**
   (1) Constructions by Zermelo-Fraenkel axioms: set builder notation (1.1), Cartesian product (1.2), subsets (1.3), power set (1.4), union, intersection, difference (1.5), complement (1.6), arbitrary unions and intersections (1.8)
   (2) Proving identities for sets: laws of set operations, Venn diagrams (1.7), proving $a \in A$, $A \subseteq B$, $A = B$ (see also 8.1-8.3).

2. **Logic.**
   (1) Statements: logical connectives (2.2), if (2.3), truth tables (2.5), logical equivalence (2.6), iff (2.4), negations (2.10)
   (2) Quantifiers (2.7): negating quantified statements, checking whether quantified statement is true (proof for universally quantified statement, example for existentially quantified statement)

3. **Counting.**
   (1) Lists: with/without repetitions, permutations - factorials (3.1-3.4), subsets - binomial coefficients (3.5), integer solutions of $x_1 + x_2 + \cdots + x_n = k$ (3.8)
   (2) Inclusion-Exclusion (3.7)

4. **Modular arithmetic.**
   (1) Integers: divisibility, division algorithm, gcd, lcm (4.2, 7.3), Euclidean algorithm (handout) Bezout's identity and coefficients (Prop 7.1), fundamental theorem of arithmetic (10.4)
   (2) congruences (5.2), integers mod n (11.5)

5. **Proof methods.**
   (1) direct proof (4.2, 4.3), contrapositive proof (5), proof by contradiction (6), proof of if-and-only-if statements
   (2) induction (10.1, 10.2)

6. **Relations and functions.**
   (1) Relations: reflexive, symmetric, antisymmetric (handout), transitive (11.2), equivalences (11.3), classes and partitions (11.4)
   (2) Functions: domain, codomain (12.1), image, pre-image (12.6), injective, surjective, bijective (12.2), pigeonhole principle (12.3), composition (12.4), inverse functions (12.5)