PROOF STRATEGIES

PETER MAYR (MATH 2001, CU BOULDER)

1. Conditional statements

How to prove "If P, then Q":

(1) **Direct proof:**

Assume P holds. Show that Q holds.

(2) **Contrapositive proof:**

Assume $\sim$ Q. Show $\sim$ P.

Then you know $\sim Q \Rightarrow \sim P$ which is logically equivalent to $P \Rightarrow Q$.

(3) **Proof by contradiction**:

Assume P $\wedge \sim$ Q. Show this implies FALSE (a contradiction).

(see below)

2. Biconditional statements

How to prove "P if and only if Q": Prove "If P, then Q" and "If Q, then P".

3. Proof by contradiction

Instead of proving some statement P directly:

Assume its negation $\sim$ P holds. Show this implies FALSE (a contradiction to something known to be true).

Then $\sim$ P must have been FALSE. Hence P is TRUE.

4. Induction

To show that a statement S_n is true for every $n \in \mathbb{N}$:

(1) **basis step:** Show S_1.

(2) **inductive step:** Show $S_k \Rightarrow S_{k+1}$ for any $k \in \mathbb{N}$.

For **strong induction** the induction step is instead:

Show $(S_1 \wedge \cdots \wedge S_k) \Rightarrow S_{k+1}$ for any $k \in \mathbb{N}$.

Date: March 23, 2018.