NEGATING STATEMENTS

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When asked to negate a statement like 'If P, then Q', it is not enough to just state 'It is not true that if P, then Q'. You need to simplify the negated statement as far as possible to see what it means.

Example 1. *Simplify for statements P, Q:*

(1) $\sim (P \wedge Q) =$
(2) $\sim (P \vee Q) =$
(3) $\sim (P \Rightarrow Q) =$
(4) $\sim (P \Leftrightarrow Q) =$
(5) $\sim (\forall x \in A: \ P(x)) =$
(6) $\sim (\exists x \in A: \ P(x)) =$

When negating a concrete example, first determine which of the above forms it has. Rewrite it in symbolic logic (using quantifiers, $\wedge, \vee, \sim, \Rightarrow$ if necessary). Then it becomes easy to negate it using the above rules.

Example 2. *Negate:*

(1) *If x is even, then x^2 is even.*
(2) *x is even only if $\frac{x}{2}$ is an integer.*
(3) *If x is odd and y is odd, then $x + y$ is even.*
(4) *x is even iff x^2 is even.*
(5) *For every even integer x, also x^2 is even.*
(6) *For every prime p there is a prime q with $q > p$.*
(7) *There exists $n \in \mathbb{N}$ such that $n > p$ for all primes p.*