

Math 2001 - Writing project 2

First draft due October 28, **final draft November 2, 2020**

The following **writing project** will be graded on clarity and correctness and should be typed in LaTeX.

Problem. There are 10 coins all of the same weight with the exception of one fake coin that is either lighter or heavier than the others (you do not know which). Given an balance scale you can compare the weight of one set of coins with another. Show that using the scale three times is enough to find the fake coin. Bonus question: Can you do it by using the scale twice?

Your write up should include the following:

- (1) A section describing the problem.
- (2) A theorem stating the main result in a few sentences, e.g.

Given 10 coins, one of which is fake (lighter or heavier than the others), the fake one can be determined using a balance scale 3 times or less.

- (3) A proof of the theorem describing the strategy for achieving the result. Try to organize the case distinction as clearly as possible, e.g:

Proof. Number coins $1, 2, 3, \dots, 10$. First weigh coins $1, 2, 3$ against $4, 5, 6$.

- Case 1, coins $1, 2, 3$ weigh the same as $4, 5, 6$: Then the fake is among \dots
- Case 2, coins $1, 2, 3$ are lighter than $4, 5, 6$: Then $7, 8, 9, 10$ are real. Weigh $1, 2, 3$ against $7, 8, 9$.
 - Case 2.1, coins $1, 2, 3$ weigh the same as $7, 8, 9$: Then the fake is among \dots and heavier because \dots
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Bonus Hint. Organizing the proof as above, we see that every use of the scale yields 3 cases. How many different cases can you get after 2 weighings? Why is this not enough to determine 1 fake coin out of 10?

- (4) Give precise arguments for all your statements.