Math 2001 - Assignment 13

Due December 4, 2020

(1) Let $f\colon A \to B, g\colon B \to C$. Show that
  (a) If $g \circ f$ is injective, then f is injective.
  (b) If $g \circ f$ is surjective, then g is surjective.
  Hint: Use contrapositive proofs.
  Give examples for f, g on $A = B = C = \mathbb{N}$ such that
  (c) $g \circ f$ is injective but g is not injective;
  (d) $g \circ f$ is surjective but f is not surjective.

(2) (a) Show that

$$f : \mathbb{R} - \{1\} \to \mathbb{R} - \{2\}, x \mapsto \frac{2x+1}{x-1}$$

  is bijective.
  (b) Determine f^{-1}.

(3) Try to you find an inverse for $f\colon \mathbb{R} \to \mathbb{R}^+,\ x \mapsto e^{x^3+1}$. Is f bijective?

(4) Find the inverse for $f\colon \mathbb{R}^2 \to \mathbb{R}^2,\ (x, y) \mapsto (3x + y, x - 2y)$.

(5) Let U be a set, and let c be the function on the power set of U that maps every set to its complement in U, i.e.,

$$c\colon P(U) \to P(U), X \mapsto \bar{X}.$$

Determine c^{-1} if it exists.

(6) Give an explicit bijection $f\colon [0, 1] \to (0, 1)$. Show that your function f is bijective.
  Hint: Consider some sequence $0, 1, \ldots$ in $[0, 1]$ and use the idea of Hilbert's hotel.