Math 2001 - Assignment 11

Due November 13, 2020

(1) Define a sequence of integers $a_1 := 1, a_2 := 1$ and

$$a_n := 2a_{n-1} + a_{n-2} \text{ for } n \geq 3.$$

Prove that a_n is odd for all $n \in \mathbb{N}$ by strong induction.

(2) Prove or disprove that the following relations are reflexive, symmetric, antisymmetric, transitive. Which are equivalences, which partial orders?
   (a) $\neq$ on $\mathbb{Z}$
   (b) $\subseteq$ on the power set $P(A)$ of a set A

(3) Prove or disprove that the following relations are reflexive, symmetric, antisymmetric, transitive. Which are equivalences, which partial orders?
   (a) $|$ (divides) on $\mathbb{N}$
   (b) $R = \{(x, y) \in \mathbb{R} \colon |x - y| \leq 1\}$

(4) List the equivalence classes for these equivalence relations:
   (a) The relation $\sim$ on subsets A, B of $\{1, 2, 3\}$ where $A \sim B$ if $|A| = |B|$.
   (b) $R = \{(x, y) \in \mathbb{Z} \colon |x| = |y|\}$ on $\mathbb{Z}$

(5) (a) Given finite sets A and B. How many different relations are there from A to B?
   (b) How many different equivalence relations are there on $A = \{1, 2, 3\}$? Describe them all by listing the different partitions of A.

(6) Let $\sim$ be an equivalence relation on a set A, let $a, b \in A$. Let $[a]$ denote the equivalence class of a modulo $\sim$. Show that

$$a \not\sim b \text{ iff } [a] \cap [b] = \emptyset.$$