

Math 2001 - Assignment 10

Due November 6, 2020

The first 4 problems are meant to be revision for the midterm. Do them before Wednesday.

- (1) (a) How many permutations of the alphabet a, \dots, z contain the word “fish”?
- (b) How many permutations of the alphabet do not contain any of the words “fish”, “rat” or “bird”?
- (2) A regular poker card set has 4 suits and 13 cards for each suit.
 - (a) How many sets of 5 cards (out of 52) are there with 4 cards of one kind?
 - (b) How many sets of 5 cards (out of 52) are there with all cards of the same suit?
- (3) [1, Chapter 10, exercise 8] Show that for every $n \in \mathbb{N}$:

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

- (4) Show by induction that for every natural number $n \geq 4$:

$$2^n \geq n^2$$

- (5) Let p_1, p_2, \dots denote the list of all primes. Show that for integers $a = \prod_{i \in \mathbb{N}} p_i^{e_i}$, $b = \prod_{i \in \mathbb{N}} p_i^{f_i}$ with $e_i, f_i \in \mathbb{N}_0$ for $i \in \mathbb{N}$,

$$\text{lcm}(a, b) = \prod_{i \in \mathbb{N}} p_i^{\max(e_i, f_i)}.$$

- (6) Show for all $a, b \in \mathbb{N}$:

$$\text{gcd}(a, b) \cdot \text{lcm}(a, b) = ab$$

Hint: Use the formula for gcd and lcm from class and the previous problem.

REFERENCES

- [1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: <http://www.people.vcu.edu/~rhammack/BookOfProof/>