Math 2001 - Assignment 9

Due October 30, 2020

(1) [1, Chapter 6, exercise 8] Prove by contradiction: Let $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

(2) Prove for all $x, y \in \mathbb{R}$:
If x is rational and xy is irrational, then y is irrational.

(3) Compute:
   (a) $3 \cdot 4 \mod 7$
   (b) $2 - 9 \mod 11$
   (c) $2^6 \mod 9$
   (d) Solve for $x \in \mathbb{Z}$: $13x \equiv 1 \mod 31$
   Hint: First solve the equation $13x + 31y = 1$ using the extended Euclidean algorithm.

(4) Prove: Let $a, b, c, d \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \mod n$ and $c \equiv d \mod n$, then $a + c \equiv b + d \mod n$.

(5) Prove by induction that for every $q \in \mathbb{R}$ with $q \neq 1$ and for every $n \in \mathbb{N}_0$:

$$1 + q^1 + q^2 + \cdots + q^n = \frac{1 - q^{n+1}}{1 - q}$$

(6) [1, Chapter 10, exercise 2] Show by induction that for every $n \in \mathbb{N}$:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`