Math 2001 - Assignment 8

Due October 23, 2020

(1) Compute $\gcd(a, b)$ and its Bezout coefficients using the Euclidean algorithm for the following numbers. Then find $\text{lcm}(a, b)$.
   (a) $a = 85, b = 25$ (b) $a = 57, b = 24$

(2) Solve the following for $u, v \in \mathbb{Z}$:
   (a) $33u + 10v = -5$ (b) $44u + 10v = 5$

(3) Let $a, b, c \in \mathbb{Z}$ with a, b not both 0. Show that

$$\exists x, y \in \mathbb{Z}\colon\ x \cdot a + y \cdot b = c \text{ iff } \gcd(a, b)|c.$$

Hint: There are 2 implications to show:
   (a) If $x \cdot a + y \cdot b = c$, then $\gcd(a, b)|c$.
   (b) If $\gcd(a, b)|c$, then there are $x, y \in \mathbb{Z}$ such that $x \cdot a + y \cdot b = c$.
Use Bezout's identity!

(4) Two integers have the *same parity* if both are even or both are odd. Otherwise they have *opposite parity*.
   Let $a, b \in \mathbb{Z}$. Show that if $a + b$ is even, then a, b have the same parity.
Hint: Use a contrapositive proof.

(5) Show for all $a \in \mathbb{Z}$: If a^2 is even, then a is even.
Hint: Which type of proof is the best to use?

(6) Complete the following proof of **Euclid's Lemma:**
   Let p be a prime, $a, b \in \mathbb{Z}$. If $p|ab$, then $p|a$ or $p|b$.

Proof: Assume ______________ but $p \nmid a$. We will show $p|b$.
   By Bezout's identity we have $u, v \in \mathbb{Z}$ such that

________________________ $= \gcd(a, p)$.

Since p is ______________ and $p \nmid a$, we have $\gcd(a, p) =$_____.
Hence

$ua + vp =$ _____.

Multiplying this equation by _____ yields

________________________ $= b$

Since $p|$______________ and $p|$______________, we have a multiple of p on the left hand side of this equation. Thus __________.
□

Please hand in this sheet of paper with your solution of 6.