Math 2001 - Assignment 7

Due October 16, 2020

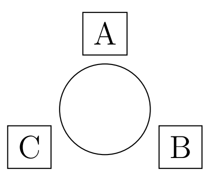
Be careful to write down every step in the proofs of problems 5,6 and reduce every statement to definitions or other statements that were already proved in class.

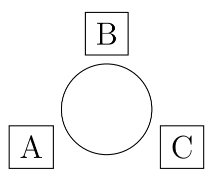
(1) (a) In how many ways can you put 7 identical balls into 3 rooms so that every room contains at least one ball?
    (b) [1, Section 3.8]: Exercise 14

(2) [1, Section 3.8]: Exercises 4 and 5

(3) How many 4-letter "words" can you form from the alphabet $A, \ldots, Z$ if the letters are in alphabetical order and
    (a) repetitions are not allowed, e.g., BEFS.
    Hint: Such a word is uniquely determined by which letters from $A, \ldots, Z$ occur in it. Which of our standard models for counting lists applies?
    (b) repetitions are allowed, e.g., BFFS.
    Hint: Every such word is uniquely determined by how often each letter from $A, \ldots, Z$ occurs in it.

(4) (a) In how many different ways can you line up n people?
    (b) In how many different ways can you line up n people in a circle?
    Hint: Since a circle has no beginning or end, two arrangements are the same if one is obtained from the other by rotation, e.g., the following are considered equal:





    Represent arrangements by lists. When do 2 lists describe the same arrangement?

(5) $a \in \mathbb{Z}$ is *even* if $a = 2n$ for some $n \in \mathbb{Z}$.
    $a \in \mathbb{Z}$ is *odd* if $a = 2n+1$ for some $n \in \mathbb{Z}$.
    Prove:
    (a) Let $a \in \mathbb{Z}$. If a is even, then a^2 is even.
    Hint: Use a direct proof. Assume a is a multiple of 2. Show that a^2 is a multiple of 2.
    (b) If x is an odd integer, then 8 divides $x^2 - 1$.
    Hint: Use a direct proof. Then split into 2 cases.

(6) Complete the proof from class that $\gcd(a, b) = \gcd(a - qb, b)$ for all $a, b, q \in Z$ with not both a and b equal 0.
    Assume $d|a - qb$ and $d|b$. Show that $d|a$ and $d|b$.

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`