

Math 2001 - Assignment 5

Due October 2, 2020

Problems 1-3 are review material for the first midterm on Sets and Logic September 30. So you should solve them before Wednesday!

(1) Simplify:

(a) $\bigcup_{i=0}^4 [i, 2i + 1]$

(b) $\bigcap_{n \in \mathbb{N}} \{x \in \mathbb{Z} : x \geq n\}$

(c) $\bigcup_{x \in [0,1]} \{x\} \times [1, 2]$

(d) $\bigcup_{x \in [0,1]} \{x\} \times [0, x]$

(2) (a) Is it true that for all statements P, Q, R :

$$(P \Rightarrow Q) \wedge P = Q$$

Prove it or give a counter-example.

(b) Show the distributive law $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$.

(3) Write using quantifiers and logical operations. Is the statement true? Give its negation.

(a) The square of any real number is non-negative.

(b) There exists an integer x such that $x^y = x$ for all integers y .

(c) For all reals x and y we have that $xy = 0$ implies $x = 0$.

(4) How many lists of length 4 are there with entries from A, \dots, Z if

(a) repetition is allowed,

(b) repetition is not allowed,

(c) repetition is not allowed and the list must contain A,

(d) repetition is allowed and the list must contain A.

(5) [1, Section 3.3]: Exercise 2

(6) How many standard Colorado license plates (3 numbers followed by 3 letters) have at least one number or letter repeated?

REFERENCES

- [1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: <http://www.people.vcu.edu/~rhammack/BookOfProof/>