Math 2001 - Assignment 4

Due September 25, 2020

(1) (a) How many different truthtables (Boolean functions) are
    there for 2 statements x_1, x_2? How many for k statements
    $x_1, \ldots, x_k$?

(b) Let $f(x_1, x_2, x_3)$ be a Boolean function that is true for the
    following assignments and false otherwise.

x_1	x_2	x_3	$f(x_1, x_2, x_3)$
T	T	F	T
T	F	T	T
F	T	T	T

Write an expression for $f(x_1, x_2, x_3)$ using only $\wedge, \vee, \sim$.

(2) [1, Section 2.7]: Exercises 4,6,7,9,10. Also give the negation of
    the corresponding statements.

(3) Formulate the following sentences using quantifiers and logical
    operations. Are they true? Negate them.

    (a) For all integers n we have that $n(n+1)$ is even.

    (b) There exists a real number z such that $x + z = x$ for every
        real x.

    (c) Every real number is smaller than some integer.

(4) Negate the following sentences. Are they true?

    (a) If x^2 is rational, then so is x.

    (b) $xy = 0$ iff $x = 0$ or $y = 0$

    (c) The derivative of a polynomial function f is 0 iff f is con-
        stant.

    (d) $\exists x \in \mathbb{R} : x^2 = -1$

    (e) $\forall r \in \mathbb{R} : \sin(r\pi) = 0 \Leftrightarrow r$ is an integer

(5) True or false? Give a proof or a counter-example:

    (a) $\forall x \in \mathbb{R} \; \forall y \in \mathbb{R} \; x + y = 1$

    (b) $\forall x \in \mathbb{R} \; \exists y \in \mathbb{R} \; x + y = 1$

    (c) $\exists x \in \mathbb{R} \; \forall y \in \mathbb{R} \; x + y = 1$

    (d) $\exists x \in \mathbb{R} \; \exists y \in \mathbb{R} \; x + y = 1$

(6) Write as complete English sentences. True or false? Negate.

    (a) $\forall a \in \mathbb{R} \; \exists b \in \mathbb{R} \; \forall c \in \mathbb{R} : \; a < b \Rightarrow c < b$

    (b) $\forall$ set $A \; \forall$ set $B \; \exists$ set $C : \; A \cup B = C$.

    (c) $\forall x, y \in \mathbb{R} \; \forall \varepsilon > 0 \; \exists \delta > 0 : |x - y| < \varepsilon \Rightarrow |2x - 2y| < \delta$

REFERENCES

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013.
    Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`