# Math 2001 - Assignment 3

Due September 18, 2020

(1) Simplify the following sets and justify your answers:
(a) $\bigcap_{n\in\mathbb{N}}\{nz \ : \ z \in \mathbb{Z}\}$ (b) $\bigcup_{x\in\mathbb{R}}[-x, x]$ (c) $\bigcap_{n\in\mathbb{N}}(-\frac{1}{n}, \frac{1}{n})$

(2) Are the following statements? If so, determine whether they are true or false and write down their negation.
   (a) Some swans are black.
   (b) Every real number is an even integer.
   (c) 2 is even, and 3 is even.
   (d) If $x$ is an even integer, then $x + 1$ is odd.
   (e) $2x = 1$

(3) [1, Section 2.3]: Exercises 2,3,4,5,10

(4) Are the given statements true? Formulate their negations:
   (a) Not all sides of a triangle have the same length or all its angles are equal.
   (b) If the integer $x$ is a multiple of 6, then $x$ is even.
   (c) $x \in \mathbb{R}$ is a square $\Rightarrow x \geq 0$.
   (d) $2^n + 1$ is a prime number for every $n \in \mathbb{N}$.
   (e) There exists an even prime.

(5) Use truth tables to show that the following hold for all logical statements $P, Q, R$:
   (a) $P \vee (P \wedge Q) = P$
   (b) $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$

(6) Are the following equalities true for all statements $P, Q$? Consider truth tables.
   (a) $P \Rightarrow Q \ = \sim P \vee Q$
   (b) $\sim (P \Leftrightarrow Q) \ = \ \sim P \Leftrightarrow \ Q \ = \ P \Leftrightarrow \sim Q$

## References


[1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`