Math 2001 - Assignment 2

Due September 11, 2020

(1) Describe the following using set builder notation (either Axiom of Specification or Replacement):
  (a) $A =$ the set of points in $\mathbb{R}^2$ on the line through $(2,3)$ that is parallel to the y-axis
  (b) $B =$ the set of points $(x,y) \in \mathbb{R}^2$ on the line through $(1,2)$ and $(3,4)$
  (c) $C =$ the set of points in $\mathbb{R}^2$ that lie on a circle with center $(0,0)$ and radius 2

(2) In the universe $U := \mathbb{N}$, compute for subsets $A = \{1,2,3,4,5\}$, $B = \{x \in U : x \text{ is even }\}$, and $C = \{x \in U : x \geq 4\}$:
(a) $A \cap \overline{B}$ (b) $A \cup (B \cap C)$ (c) $(A - B) \cup B$

(3) Are the following true for all sets A, B in a universe U?
  (a) $A - B = B - A$
  (b) $A \cup B \subseteq (A \cap \overline{B}) \cup (B \cap \overline{A})$
Consider Venn diagrams first and then either write a proof that the equations hold or give an example where they fail.

(4) Show that for all sets A, B, C

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

without Venn diagrams.

We already showed that the lefthand side is contained in the righthand side in class. So it only remains to write a proof for the converse inclusion,

$$(A \cup B) \cap C \supseteq (A \cap C) \cup (B \cap C).$$

(5) Show for all sets A, B in the universe U:

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad \text{(de Morgan's law)}$$

First use Venn diagrams. Then write down a proof.

(6) Read Section 1.8 in [1]. Then simplify the following sets and justify your answers:
(a) $\bigcup_{n\in\mathbb{N}}(0,n]$ (b) $\bigcap_{n=1}^{3}\{nz : z \in \mathbb{Z}\}$ (c) $\bigcup_{A\in P(\mathbb{N})} A$

In (a) we have $(0,n] = \{x \in \mathbb{R} \ : \ 0 < x \leq n\}$, the real interval from 0 to n that does not contain 0 but contains n.

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`