

Math 2001 - Assignment 1

Due January 24, 2020

- (1) Element, subset or neither? Explain for each of the following whether $A \in B$ and $A \subseteq B$ are true or false:

(a) $A = 4, B = \mathbb{Z}$ (b) $A = \{1, 2, 3\}, B = \mathbb{Z}$

(c) $A = \{1, 2\}, B = \{1, 2, \{1, 2\}\}$

Solution

(a) $4 \in \mathbb{Z}$ but $4 \not\subseteq \mathbb{Z}$

(b) $\{1, 2, 3\} \notin \mathbb{Z}$ but $\{1, 2, 3\} \subseteq \mathbb{Z}$

(c) $\{1, 2\} \in \{1, 2, \{1, 2\}\}$ since $\{1, 2\}$ is the 3rd element in $\{1, 2, \{1, 2\}\}$ and $\{1, 2\} \subseteq \{1, 2, \{1, 2\}\}$ since $1, 2 \in \{1, 2, \{1, 2\}\}$

- (2) Are the following true for $A = \{1, \{2, 3\}\}$ or not?

(a) $\{2, 3\} \in A$ (b) $\{2, 3\} \subseteq A$ (c) $\emptyset \in A$ (d) $|A^2| = 9$.

Solution

(a) True (b) False (c) False (d) False, $|A^2| = 4$

- (3) [1, Section 1.1]: Exercises 1,12,15,29,38

Solution

1. $\{5x - 1 : x \in \mathbb{Z}\} = \{\dots, -6, -1, 4, 9, \dots\}$

12. $\{x \in \mathbb{Z} : |2x| < 5\} = \{-2, -1, 0, 1, 2\}$

15. $\{5a + 2b : a, b \in \mathbb{Z}\} = \mathbb{Z}$

29. $|\{\{1\}, \{2, \{3, 4\}\}, \emptyset\}| = 3$

38. $|\{x \in \mathbb{N} : 5x \leq 20\}| = |\{1, 2, 3, 4\}| = 4$

- (4) Write each of the following sets using a defining property (Axiom of Specification) and using a function (Axiom of Replacement):

(a) $A = \{\dots, -8, -4, 0, 4, 8, 12, \dots\}$

(b) $B = \{0, 1, 2\}$

(c) $C =$ the set of even squares

Solution

 First specification, then replacement:

$$A = \{x \in \mathbb{Z} : x \text{ is a multiple of } 4\} = \{4x : x \in \mathbb{Z}\}$$

$$B = \{x \in \mathbb{Z} : 0 \leq x \leq 2\} = \{x : x \in \{0, 1, 2\}\}$$

(the second option for B is technically correct though it looks a bit like cheating)

$$C = \{x \in \mathbb{Z} : x \text{ is an even square}\} = \{(2x)^2 : x \in \mathbb{Z}\}$$

- (5) Let $A = \{0, 1\}$ and $B = \{a, b, c\}$. Enumerate the elements of the following sets:

(a) $B \times A$

(b) $A \times \emptyset$

(c) A^3

Solution

(a) $B \times A = \{(a, 0), (a, 1), \dots, (c, 1)\}$

(b) $A \times \emptyset = \emptyset$

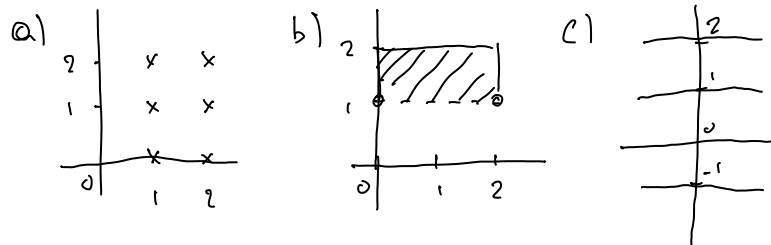
(c) $A^3 = \{(0, 0, 0), \dots, (1, 1, 1)\}$ has 8 elements of triples with 0,1.

- (6) Sketch the following Cartesian products in the plane \mathbb{R}^2 . Be careful to denote whether the boundaries of your figures are contained in the sets or not (Use dashed lines for boundaries that are not included, solid lines for boundaries that are included).

(a) $\{1, 2\} \times \{0, 1, 2\}$

(b) $[0, 2] \times (1, 2]$

(c) $\mathbb{R} \times \mathbb{Z}$

**REFERENCES**

- [1] Richard Hammack. The Book of Proof. Creative Commons, 3rd edition, 2018. Available for free: <http://www.people.vcu.edu/~rhammack/BookOfProof/>