

COMBINATORICS

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1. Combinatorial functions:

$n! := n \cdot (n-1) \cdots 2 \cdot 1$ is the number of permutations of $\{1, \dots, n\}$.

$$\binom{n}{k} := \frac{n!}{k!(n-k)!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

2. Counting the most common objects:

A k -list is a list of length k .

A k -subset is a set of size k .

Number of lists of length k with entries in $\{1, \dots, n\}$

| | with repetition | without repetition |
|------------------------------|---|--------------------------------|
| order matters | n^k k -lists | $\frac{n!}{(n-k)!}$ |
| order does not matter | $\binom{n+k-1}{k}$ k -multisubsets | $\binom{n}{k}$ k -subsets |

3. Inclusion-Exclusion:

For finite sets A_1, \dots, A_n ,

$$\begin{aligned}
 |A_1 \cup \dots \cup A_n| &= |A_1| + \dots + |A_n| \\
 &\quad - |A_1 \cap A_2| - |A_1 \cap A_3| - \dots - |A_{n-1} \cap A_n| \\
 &\quad + |A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \dots + |A_{n-2} \cap A_{n-1} \cap A_n| \\
 &\quad \vdots \\
 &\quad + (-1)^{n-1} |A_1 \cap A_2 \cap \dots \cap A_n|
 \end{aligned}$$