Math 2001 - Assignment 13

Due December 2, 2016

(1) (a) Show that

$$f: \mathbb{R} - \{1\} \to \mathbb{R} - \{2\}, x \mapsto \frac{2x+1}{x-1}$$

is bijective.

- (b) Determine f^{-1} .
- (2) Try to you find an inverse for $f \colon \mathbb{R} \to \mathbb{R}^+$, $x \mapsto e^{x^3+1}$. Is f bijective?
- (3) Find the inverse for $f: \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (3x + y, x 2y)$.
- (4) Let c be the function on the power set of \mathbb{Z} that maps every set to its complement, i.e.,

$$c\colon P(\mathbb{Z})\to P(\mathbb{Z}), X\mapsto \bar{X}.$$

Determine c^{-1} .

(5) The following **writing project** is worth 10 points. It will be graded on clarity and correctness, should be typed and handed it on a separate piece of paper.

Let $m, n \in \mathbb{N}$. How many surjective functions are there from $\{1, \ldots, m\}$ onto $\{1, \ldots, n\}$?

Your write up should include the following:

- (a) Describe the problem. What is surjective?
- (b) State the main result in a few sentences.
- (c) Prove the result. Hint: inclusion-exclusion.
- (d) Give precise arguments for all your statements.