Math 2001 - Assignment 12

Due November 18, 2016

(1) Give the addition and multiplication tables for $\mathbb{Z}_6$.

(2) Dividing in $\mathbb{Z}_n$ means solving an equation $[a] \cdot [x] = [b]$ for $[x]$.
   Solve $[8] \cdot [x] = [1]$ in $\mathbb{Z}_{37}$.
   Hint: Use the Euclidean algorithm to solve $8x \equiv 1 \mod 37$.

(3) Let p be a prime. Show that for every $[a] \in \mathbb{Z}_p$ with $[a] \neq [0]$ there exists $[x] \in \mathbb{Z}_p$ such that

$$[x] \cdot [a] = [1].$$

   Is the same statement true if p is not prime?

(4) Let $f = \{(x, 4x+1) \mid x \in \mathbb{Z}\}$ be a function on $\mathbb{Z}$.
   (a) Give domain, codomain, and range of f. What is $f(9)$?
   (b) Is f one-to-one, onto, bijective?

(5) Give examples for
   (a) a function $f \colon \mathbb{Z} \to \mathbb{Z}$ that is not injective but surjective;
   (b) a function $g \colon \{1, 2, 3\} \to \{1, 2\}$ that is neither injective nor surjective;
   (c) a bijective function $h \colon \{1, 2, 3\} \to \{1, 2\}$.

(6) Let A, B be finite sets. How many functions from A to B are there?
   How many bijective functions from A to B?