# Math 2001 - Assignment 11

Due November 11, 2016

(1) Prove or disprove that the following relations are reflexive, symmetric, antisymmetric, transitive:
  (a) $<$ on $\mathbb{Z}$
  (b) $\neq$ on $\mathbb{Z}$
  (c) $\subseteq$ on the power set $P(A)$ of a set $A$

(2) Prove or disprove that the following relations are reflexive, symmetric, antisymmetric, transitive:
  (a) $|$ (divides) on $\mathbb{N}$
  (b) $R = \{(x, y) \in \mathbb{R} \colon |x - y| \leq 1\}$
  (c) $S = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2)\}$ on $\{1, 2, 3\}$

(3) List the equivalence classes for these equivalence relations:
  (a) The relation "has the same size" on the power set of $\{1, 2, 3\}$
  (b) $\equiv_n$ on $\mathbb{Z}$
  (c) $R = \{(x, y) \in \mathbb{Z} \colon |x| = |y|\}$ on $\mathbb{Z}$

(4) How many different equivalence relations are there on $A = \{1, 2, 3\}$? Describe them all by listing their equivalence classes.

(5) Given finite sets $A$ and $B$. How many different relations are there from $A$ to $B$?

(6) Let $\sim$ be an equivalence relation on a set $A$, let $a, b \in A$. Let $[a]$ denote the equivalence class of $a$ modulo $\sim$. Show that

$$a \not\sim b \text{ iff } [a] \cap [b] = \emptyset.$$

## References


[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`