Math 2001 - Assignment 10

Due November 4, 2016

(1) Prove by induction that for every $q \in \mathbb{R}$ with $q \neq 1$ and for every $n \in \mathbb{N}_0$:
$$1 + q^1 + q^2 + \cdots + q^n = \frac{1 - q^{n+1}}{1 - q}$$

(2) [1, Chapter 10, exercise 8] Show that for every $n \in \mathbb{N}$:
$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

(3) Let $n \in \mathbb{N}$ and let $A_1, \ldots, A_n$ be sets in some universe U. Show by induction that
$$\overline{A_1 \cup A_2 \cup \cdots \cup A_n} = \bar{A}_1 \cap \bar{A}_2 \cap \ldots \bar{A}_n$$

Hint: for the base case consider $n = 1$ and $n = 2$.

(4) Show for all $a, b \in \mathbb{N}$:
$$\gcd(a, b) \cdot \operatorname{lcm}(a, b) = ab$$

(5) The following **writing project** is worth 10 points. It will be graded on clarity and correctness, should be typed and handed it on a separate piece of paper.

Towers of Hanoi: How many moves are necessary to move n disks from one place to another?

Your write up should include the following:
(a) Describe the problem.
(b) State the main result in a few sentences.
(c) Prove the result by induction.
(d) Give precise arguments for all your statements.

REFERENCES

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`