Math 2001 - Assignment 7

Due October 14, 2016

Be careful to write down every step in the proofs of 3,4 and reduce every statement to definitions or other statements that were already proved in class.

(1) How many different seating arrangements are there on a round table with n seats?

Since a round table has no beginning or end, two arrangements are the same if one is obtained from the other by rotation, e.g., the following are considered equal:

(2) Compute $\gcd(a, b)$ and the Bezout coefficients using the Euclidean algorithm for the following numbers. Then find $\text{lcm}(a, b)$.
   (a) $a = 85, b = 25$
   (b) $a = 57, b = 24$

(3) Show that if x is an odd integer, then 8 divides $x^2 - 1$.
Recall: x is odd if it is of the form $x = 2y + 1$ for some $y \in \mathbb{Z}$.

(4) Let x be an integer. Show that if x^2 is odd, then x is odd.

(5) The following **writing project** is worth 10 points. It will be graded on clarity and correctness, should be typed and handed it on a separate piece of paper.

There are 10 coins all of the same weight with the exception of one fake coin that is either lighter or heavier than the others (you do not know which). Given an balance scale you can compare the weight of one set of coins with another. Show that using the scale three times is enough to find the fake coin. Can you do it by using the scale twice?

Your write up should include the following:
   (a) Describe the problem.
   (b) State the main result in a few sentences.
   (c) What is the strategy for achieving the result?
   (d) Give precise arguments for all your statements.