

Math 2001 - Assignment 5

Due September 30, 2016

- (1) Negate the following sentences:
 - (a) $\exists x \in \mathbb{R} : x^2 = -1$
 - (b) $\forall r \in \mathbb{R} : \sin(r\pi) = 0 \Leftrightarrow r$ is an integer
 - (c) There exists a right triangle that is not isosceles.
 - (d) $\forall x \in \mathbb{R}^+ \exists n \in \mathbb{N} : \frac{1}{n} \leq x$.
 - (e) $\exists m \in \mathbb{N} \forall p \in \mathbb{N} : p \text{ prime} \Rightarrow p \leq m$
- (2) How many lists of length 4 are there with entries from A, \dots, Z if
 - (a) repetition is allowed,
 - (b) repetition is not allowed,
 - (c) repetition is not allowed and the list must contain A,
 - (d) repetition is allowed and the list must contain A.
- (3) [1, Section 3.1]: Exercise 4
- (4) How many standard Colorado license plates (3 numbers followed by 3 letters) have at least one number or letter repeated?
- (5) How many different 5-card hands form a Full House? Pick 5 cards from a standard 52-card deck such that 3 are of a kind and the remaining 2 are of a kind.
- (6) Give 2 reasons why the following is true for all $k, n \in \mathbb{N}$ with $0 \leq k \leq n$:

$$\binom{n}{k} = \binom{n}{n-k}$$

REFERENCES

- [1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: <http://www.people.vcu.edu/~rhammack/BookOfProof/>