

Math 2001 - Assignment 2

Due September 9, 2016

- (1) Describe the following using set builder notation:
- (a) $A =$ the set of points in \mathbb{R}^2 on the line through $(2, 3)$ that is parallel to the y -axis
 - (b) $B =$ the set of points $(x, y) \in \mathbb{R}^2$ on the line through $(1, 2)$ and $(3, 4)$
 - (c) $C =$ the set of points in \mathbb{R}^2 that lie on a circle with center $(0, 0)$ and radius 2
- (2) For $U := \{x \in \mathbb{Z} : 1 \leq x \leq 8\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{x \in U : x \text{ is even}\}$, and $C = \{x \in U : x \geq 4\}$ compute:
- (a) $A \cap C_U(B)$
 - (b) $A \cup (B \cap C)$
 - (c) $(A - B) \cup B$
- (3) Simplify and justify your answers:
- (a) $\bigcup_{n \in \mathbb{N}} (0, n]$
 - (b) $\bigcap_{n=1}^3 \{nz : z \in \mathbb{Z}\}$
 - (c) $\bigcup_{A \in P(\mathbb{N})} A$
- (4) Are the following true for all sets A, B in a universe U ?
- (a) $A - B = B - A$
 - (b) $A \cup B \subseteq (A \cap \bar{B}) \cup (B \cap \bar{A})$

Consider Venn diagrams first and then either write a proof that the equations hold or give an example where they fail.

- (5) Show that for all sets A, B, C

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

without Venn diagrams.

Recall that we already showed that the lefthand side is contained in the the righthand side. So it only remains to write a proof for the converse,

$$(A \cup B) \cap C \supseteq (A \cap C) \cup (B \cap C).$$

- (6) Show for all sets A, B in the universe U :

$$\overline{A \cup B} = \bar{A} \cap \bar{B} \quad (\text{de Morgan's law})$$

First use Venn diagrams. Then write down a proof.