Math 2001 - Assignment 13

Due December 4, 2015

(1) Let A, B be finite sets. How many functions from A to B are there? How many bijective functions from A to B?

(2) (a) Read the proof of Theorem 12.2 in [1].

(b) Find an example of functions $f\colon A \to B, g\colon B \to C$ where g is not injective but $g \circ f$ is injective.

(3) Let $f\colon A \to B, g\colon B \to C$. Show that if $g \circ f$ is injective, then f is injective.

(Hint: use contraposition)

(4) Show that

$$f : \mathbb{R} - \{1\} \to \mathbb{R} - \{2\},\ x \mapsto \frac{2x+1}{x-1}$$

is bijective.

Recall that $\mathbb{R} - \{1\}$ is the set of all real numbers except 1.

(5) Determine f^{-1} for f from the previous exercise.

(6) Let c be the function on the power set of $\mathbb{Z}$ that maps every set to its complement, i.e.,

$$c\colon P(\mathbb{Z}) \to P(\mathbb{Z}), X \to \bar{X}.$$

Determine c^{-1}.

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`