Math 2001 - Assignment 12

Due November 20, 2015

(1) Let $\sim$ be an equivalence relation on a set A, let $a, b \in A$. Show that
$$a \not\sim b \text{ iff } [a] \cap [b] = \emptyset.$$
(2) Give the addition and multiplication tables for $\mathbb{Z}_6$.
(3) Dividing in $\mathbb{Z}_n$ means solving an equation $[a] \cdot [x] = [b]$ for $[x]$.
Solve $[8] \cdot [x] = [1]$ in $\mathbb{Z}_{37}$.
Hint: Use the Euclidean algorithm to solve $8x \equiv 1 \mod 37$.
(4) Let p be a prime. Show that for every $[a] \in \mathbb{Z}_p$ with $[a] \neq [0]$ there exists $[x] \in \mathbb{Z}_p$ such that
$$[x] \cdot [a] = [1].$$
Is the same statement true if p is not prime?
(5) Let $f = \{(x, 4x + 1) \mid x \in \mathbb{Z}\}$ be a function on $\mathbb{Z}$.
  (a) Give domain, codomain, and range of f. What is $f(9)$?
  (b) Is f one-to-one, onto, bijective?
(6) Give examples for
  (a) a function $f \colon \mathbb{Z} \to \mathbb{Z}$ that is not injective but surjective;
  (b) a function $g \colon \{1, 2, 3\} \to \{1, 2\}$ that is neither injective nor surjective;
  (c) a bijective function $h \colon \{1, 2, 3\} \to \{1, 2\}$.