Math 2001 - Assignment 10

Due November 6, 2015

(1) Prove for all $a, b \in \mathbb{Z}$:
$a \equiv b \mod 6$ iff $a \equiv b \mod 2$ and $a \equiv b \mod 3$
Hint: You have to consider implications in both directions.

(2) Prove for all $a, b, c \in \mathbb{Z}$: If $a \nmid bc$, then $a \nmid b$ and $a \nmid c$.

(3) Prove for all $x, y \in \mathbb{R}$:
If x is rational and xy is irrational, then y is irrational.

(4) Prove by induction that for every $q \in \mathbb{R}$ with $q \neq 1$ and for every $n \in \mathbb{N}_0$:

$$1 + q^1 + q^2 + \cdots + q^n = \frac{1 - q^{n+1}}{1 - q}$$

(5) Show that for every natural number $n \geq 4$:

$$2^n \geq n^2$$

(6) [1, Chapter 10, exercise 8] Show that for every $n \in \mathbb{N}$:

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`