Math 2001 - Assignment 9

Due October 30, 2015

(1) Compute:
(a) $3 \cdot 4 \mod 7$ (b) $2 - 9 \mod 11$ (c) $2^6 \mod 9$
(d) Solve for $x \in \mathbb{Z}$: $13x \equiv 3 \mod 31$
Hint for (d): First solve the equation $13x + 31y = 3$ using the extended Euclidean algorithm.

(2) Prove: Let $a, b, c, d \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv b \mod n$ and $c \equiv d \mod n$, then $a + c \equiv b + d \mod n$.

(3) Prove by contradiction: $4x^2 - y^2 = 1$ has no integer solutions for x and y.
Hint: Factor $4x^2 - y^2$.

(4) [1, Chapter 6, exercise 8] Prove by contradiction: Let $a, b, c \in \mathbb{Z}$. If $a^2 + b^2 = c^2$, then a or b is even.

(5) [1, Chapter 10, exercise 2] Show by induction that for every $n \in \mathbb{N}$:
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(6) Prove by induction that $3 \mid n^3 - n$ for every natural number n.

References

[1] Richard Hammack. The Book of Proof. Creative Commons, 2nd edition, 2013. Available for free: `http://www.people.vcu.edu/~rhammack/BookOfProof/`