

Math 2001 - Assignment 7

Due October 16, 2015

Be careful to write down every step in the proofs of 3,4,5 and reduce every statement to definitions or other statements that were already proved in class.

- (1) n students form a line in front of the cafeteria.
 - (a) How many different lines are there with 2 students Alice and Bob next to each other?
 - (b) How many different lines are there with Alice and Bob not next to each other?
- (2) How many different seating arrangements are there on a round table with n seats?

Since a round table has no beginning or end, two arrangements are the same if one is obtained from the other by rotation, e.g., the following are considered equal:



- (3) Show that if x is an odd integer, then 8 divides $x^2 - 1$.
- (4) Prove: If p is prime and $0 < k < p$, then p divides $\binom{p}{k}$.
- (5) Let x be an integer. Show that if x^2 is odd, then x is odd.
- (6) Compute $\gcd(a, b)$ and the cofactors using the Euclidean algorithm for the following numbers. Then find $\text{lcm}(a, b)$.
 - (a) $a = 85, b = 25$
 - (b) $a = 57, b = 24$