Math 2001 - Assignment 6

Due October 9, 2015

Please explain your reasoning for your answers to the following problems.

I would like to discuss and compare solutions in class. Please indicate on your handin whether it is ok to use an anonymous copy of your solutions for this and the remaining problem sets for this purpose.

(1) The street map of Manhattan is a grid with avenues running North-South and streets East-West. How many different ways are there from the corner of 2nd Ave and C Street to the corner of 9th Ave and F Street if you only ever go South and West (never North or East).

(2) In a freshman class of 300 students 145 take English, 155 take Calculus and 120 Discrete Math. 90 take English and Calculus, 80 English and Discrete Math, 75 Calculus and Discrete Math, 60 take all three.

How many students take none of these three classes?

(3) How many positive integers less or equal 100 are not multiples of 2 or 3 or 5?

(4) In how many ways can one arrange 8 rooks on an 8×8 chessboard such that they do not attack each other?

(5) How many permutations of $\{1, 2, 3, 4, 5\}$ have 1 in the first position? How many permutations do not have 1 in first or 2 in second position?

(6) Use the Binomial Theorem to show for all $n \in \mathbb{N}$:
   (a) $\sum_{k=0}^{n} \binom{n}{k} = 2^n$
   (b) $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n} = 0$